

A Solution to the Gettier Problem

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Problem cases by Edmund Gettier¹ and others², intended to undermine the sufficiency of the three traditional conditions for knowledge, have been discussed extensively in the philosophical literature. But I believe they suffer from heretofore unnoticed flaws that undermines their effectiveness.

The key to Gettier's examples is the principle that if you are justified in holding some belief, then you are justified in accepting any statements logically implied by that belief. So, if you are justified in believing that the earth is round, then you are justified in believing that either the earth is round or Benjamin Franklin invented television. For if it is true that the earth is round, then it is also true that either the earth is round or Benjamin Franklin invented television.

In Gettier's case II, Smith is justified in believing that Jones owns a Ford. So Smith is also justified in believing that either Jones owns a Ford or Brown is in Barcelona. Yet, as it happens, Jones does not own a Ford and, unbeknownst to Smith, Brown is in Barcelona. Thus, while Smith has the true, justified belief that either Jones owns a Ford or Brown is in Barcelona, Smith has made a lucky guess and does not possess knowledge.

Or so it may seem. But let us take a closer look at the case. If it is true that Jones owns a Ford, then it is true that either Jones owns a Ford or Brown is in Barcelona. So far, so good. The sentence 'Jones owns a Ford or Brown is in Barcelona' is true if and only if the exclusive disjunction of (1), (2), and (3) obtains:

- (1) It is true that Jones owns a Ford and true that Brown is in Barcelona.
- (2) It is true that Jones owns a Ford and false that Brown is in Barcelona.
- (3) It is false that Jones owns a Ford and true that Brown is in Barcelona.

It seems obvious that the inference rule Smith follows is:

(4) P logically entails $(P \vee Q)$

But this inference rule is not consistent with the claim that $(P \vee Q)$ is true if and only if the exclusive disjunction of (1), (2), and (3) obtains. To see this, we substitute $(P \vee Q)$ in (4) with:

$[(P \wedge Q) \oplus (P \wedge \neg Q) \oplus (\neg P \wedge Q)]$

The first exclusive disjunct is equivalent to (1), the second disjunct is equivalent to (2), and the third is equivalent to (3). The result of this substitution is (5):

(5) P logically entails $[(P \wedge Q) \oplus (P \wedge \neg Q) \oplus (\neg P \wedge Q)]$

It should be obvious that (5) is not a valid rule of inference, since (5) claims that P logically entails $\neg P$. Thus, the third disjunct must be discarded and (4) becomes (6):

(6) P logically entails $P \wedge (Q \vee \neg Q)$

This implies that (3) is not among the truth conditions of 'Jones owns a Ford or Brown is in Barcelona' given that the latter is inferred from 'Jones owns a Ford'.

If Smith uses (4) as his inference rule, then the truth-conditions of his conclusion are given by (6). According to Gettier's principle Smith's belief is justified. But by the terms of the case that belief is false, since its truth conditions require that Jones own a Ford.³ So Gettier has not presented an instance in which someone has a justified, true belief but not knowledge.

There is no problem with disjunction introduction here. We expect that P logically entail $(P \vee Q)$ because P logically entails itself whether or not Q is true. That is, we expect our inference rules to be truth preserving. This is precisely the content of (6). For if P is true, then $(P \vee Q)$ is true *because* P is true – not because P is false and Q is true.

Other cases are similarly flawed. In a standard existential generalization case, Smith is justified in believing that Nogot owns a Ford⁴. So Smith is also justified in believing that someone in his office owns a Ford. As it happens Nogot does not own a Ford and, unbeknownst

to Smith, Havit does own a Ford. Thus, while Smith has the true, justified belief that someone in his office owns a Ford, he has made a lucky guess and does not possess knowledge.

If it is true that Nogot owns a Ford, then it is true that someone owns a Ford. The sentence ‘Someone in my office owns a Ford’ is true if and only if the following condition obtains⁵:

- (7) It is true that Nogot owns a Ford, or true that Havit owns a Ford, or true that Williams owns a Ford, ..., or true that N owns a Ford.

Again, it seems obvious that the inference rule Smith follows is:

- (8) Fa logically entails $\exists x(Fx)$

But this inference rule is not consistent with ‘Someone in my office owns a Ford’ being true even if Nogot does not own a Ford.

As with the previous case, we substitute $\exists x(Fx)$ in (8) with the conditions given in (7):

$$Fa \vee Fb \vee Fc \vee \dots \vee Fn$$

The result of this substitution is:

- (9) Fa logically entails $(Fa \vee Fb \vee Fc \vee \dots \vee Fn)$

Understood as (9), our rule is little more than an expanded version of (4). As with (4), the disjunction in (9) is true if and only if:

$$[(Fa \oplus \neg Fa) \wedge (Fb \oplus \neg Fb) \wedge (Fc \oplus \neg Fc) \wedge \dots \wedge (Fn \oplus \neg Fn)]$$

Substituting again, our rule becomes:

- (10) Fa logically entails $[(Fa \oplus \neg Fa) \wedge (Fb \oplus \neg Fb) \wedge (Fc \oplus \neg Fc) \wedge \dots \wedge (Fn \oplus \neg Fn)]$

Obviously, (10) is not a valid rule of inference, since it claims that Fa logically entails $\neg Fa$.

Thus, the second exclusive disjunct must be discarded and (10) becomes (11):

- (11) Fa logically entails $[Fa \wedge (Fb \oplus \neg Fb) \wedge (Fc \oplus \neg Fc) \wedge \dots \wedge (Fn \oplus \neg Fn)]$

This implies that ‘Someone in my office owns a Ford’ cannot be true if Nogot does not own a Ford, given that it is inferred from ‘Nogot owns a Ford’.

If Smith uses (8) as his inference rule, then the truth conditions of his conclusion are given by (11). He has a justified belief according to Gettier’s principle; but by the terms of the case Smith’s belief is false, since its truth conditions require that Nogot own a Ford. This is not an instance in which someone has a justified, true belief but not knowledge.

As with the disjunction case, there is no problem with existential generalization here. We expect that Fa logically entail $\exists x(Fx)$ because Fa logically entails itself whether or not there are any other substitution instances. Otherwise, our inference rule would not be truth preserving. This is precisely the content of (11). For if Fa is true, then $\exists x(Fx)$ is true *because* Fa is true and not because Fa is false and Fb is true.

Gettier’s case I suffers from the same flaw. If it is true that Jones is the person who will get the job and true that he has ten coins in his pocket, then it is true that the person who will get the job has ten coins in his pocket. The sentence ‘The man who will get the job has ten coins in his pocket’ is true if and only if the exclusive disjunction of (12) and (13) obtains:

- (12) It is true that Jones is the only person who will get the job and true that Jones has ten coins in his pocket.
- (13) It is true that X , someone other than Jones, is the only person who will get the job and it is true that X has ten coins in his pocket.

Smith follows the existential generalization rule here:

- (14) $[Fa \wedge \{(\forall y)Fy \rightarrow y = a\} \wedge Ga]$ logically entails $\exists x[Fx \wedge \{(\forall y)Fy \rightarrow y = x\} \wedge Gx]$

But again, this inference rule is not consistent with (13) being among the truth conditions of ‘The man who will get the job has ten coins in his pocket’.

By now our argument is familiar. Let Φ be the formula $[Fa \wedge \{(\forall y)Fy \rightarrow y = a\} \wedge Ga]$, let Ψ be $[Fb \dots]$, let Θ be $[Fc \dots]$, etc., for every object in the domain. When we substitute the description formula in (14) with the disjunction of formulae for all objects in the domain of quantification we have:

$$(15) \quad \Phi \text{ logically entails } (\Phi \oplus \Psi \oplus \Theta \oplus \dots \oplus \Omega)^6$$

The exclusive disjunction in (15) is true if and only if:

$$[(\Phi \oplus \neg\Phi) \wedge (\Psi \oplus \neg\Psi) \wedge (\Theta \oplus \neg\Theta) \wedge \dots \wedge (\Omega \oplus \neg\Omega)]$$

By substitution we arrive at:

$$(16) \quad \Phi \text{ logically entails } [(\Phi \oplus \neg\Phi) \wedge (\Psi \oplus \neg\Psi) \wedge (\Theta \oplus \neg\Theta) \wedge \dots \wedge (\Omega \oplus \neg\Omega)]$$

Again, (16) is not a valid rule of inference, since it claims that Φ logically entails $\neg\Phi$. Thus, the second disjunct must be discarded and (15) becomes (17):

$$(17) \quad \Phi \text{ logically entails } [\Phi \wedge \neg\Psi \wedge \neg\Theta \wedge \dots \wedge \neg\Omega]$$

This implies that (13) must also be discarded as among the truth conditions of ‘The man who will get the job has ten coins in his pocket’ given that it is inferred from ‘Jones will get the job and Jones has ten coins in his pocket’.

If Smith uses (14) as his inference rule, then the truth conditions of his conclusion are given by (17). By the terms of the case Smith’s belief is false, since his belief is true only if Jones gets the job. So Gettier has not presented an instance in which someone has a justified, true belief but not knowledge.

What about cases where Smith’s inference does not involve an invalid inference rule, or where Smith doesn’t make an inference at all? Surely those cases are still counterexamples to the traditional analysis of knowledge. For instance, Keith Lehrer⁷ gives an example wherein Smith sees Mr. Nogot driving a Ferrari and infers ‘Someone owns a Ferrari’. Smith makes this

inference “*without concluding that Mr. Nogot owns a Ferrari.*”⁸ One may reasonably assume that the lack of false lemmas in this case is evidence that Smith uses a valid inference rule.

Lehrer claims that Smith infers by existential generalization that there is someone in his class who owns a Ferrari from information gained about Mr. Nogot without reaching the intermediate conclusion that Mr. Nogot owns a Ferrari; but existential generalization simply doesn't work that way. Smith cannot *deduce* that someone in his class owns a Ferrari without a specific example of a Ferrari owner. We cannot deduce $\exists x(Fx)$ from Ga where $F \neq G$. We are left to conclude that either Smith cannot be said to have deduced that someone in his class owns a Ferrari from the fact that Mr. Nogot drives a Ferrari (in which case the former belief is unjustified); or Smith must have reached the intermediate conclusion that Mr. Nogot owns a Ferrari (which is false). Either way, this is no counterexample.

It may be objected that Lehrer's intent is that Smith's inference is inductive rather than deductive. On that reading, the following inductively supported principle justifies Smith's inference:

(18) Typically, anyone who drives a car owns it.

This principle only justifies Smith in inferring from the predicate 'drives a Ferrari' to the predicate 'owns a Ferrari'. The inductive premise does not justify the inference from 'Nogot' in 'Nogot drives a Ferrari' to 'someone' in 'Someone owns a Ferrari'. It seems that the latter inference is an existential generalization. Moreover, as a counterexample the case turns on the inference from 'Nogot' to 'someone', rather than the inference from 'drives' to 'owns', since the problem is that Nogot does not own a Ferrari but Havit does. So, this looks like an existential generalization case at heart. And my response to it is the same as to the other existential generalization cases.

Lehrer cites a second argument, one according to which counterexamples can be found of justified true beliefs that are not inferred at all. In this case, a sheep and a dog appear in Smith's visual field; but "what he takes to be a sheep is a dog, and the sheep he sees he does not take to be a sheep."⁹ We shall interpret 'I take' and 'I do not take' as indicating the existence and lack of the appropriate propositional attitude, respectively. So, "what he takes to be a sheep" is the belief:

(19) THAT is a sheep.

Conversely, "the sheep that he sees he does not take to be a sheep," indicates the lack of a belief about the other animal in Smith's visual field. Since Smith lacks the appropriate propositional attitude towards the sheep, he does not meet the first of the three conditions of the traditional analysis with respect to the sheep that he sees. Meanwhile, for (19) to be true the object denoted by 'THAT' must be a sheep; but by stipulation it is a dog. Thus, this is not an example of a true, justified belief that is not knowledge.¹⁰

What about other examples where the agent has a true belief that is not inferred, such as Goldman's barn example or Russell's stopped clock example? In Russell's stopped clock case¹¹ as modified by Scheffler¹², Alice sees a clock that says it is two o'clock. She believes it's two o'clock, and that is true. However, unknown to Alice, the clock she's looking at stopped twelve hours ago. So, she has an accidentally true, justified belief.

Some philosophers do not accept this case as a valid counterexample to the traditional analysis. Robert Shope¹³ cites concerns that this is not a case of justified belief:

For some philosophers would say that [Alice] violates a relevant procedure of rational inquiry by employing a measuring instrument that is not working, and so does not actually satisfy the intent of the [justification] condition of the standard analysis, ... This is true even if [Alice] is justified in believing that the clock is working. That justification does not carry over to the belief as to the time simply because the clock is a measuring

instrument that is not properly set up to take the measurements which [Alice] presumes that it is taking. (20)

Shope seems to argue here that two different standards of justification motivate the case. On one hand, Alice is justified in believing that the clock is working. This justification is used to claim that she has a true, justified belief. On the other hand, Alice is not justified in her belief as to the time because she employs a nonfunctional measuring instrument. This lack of justification is used to claim that *S*'s belief is accidentally true. The latter justificatory standard prevails here, since Alice's belief as to the time is at issue, not her belief as to whether or not the clock is working. Thus, the case equivocates on the justification condition, and may be rejected on that ground.

I suspect that other cases commit similar equivocations. In Goldman's case,¹⁴ Henry points to a real barn in a district full of papier-mâché facsimiles of barns and says, "That's a barn." Here, the information that the district is full of facsimile barns implies a high probability that any randomly picked barn-like object that Henry points to will be a facsimile. Thus, Henry's pointing to an actual barn is accidental. Henry does not have knowledge, in spite of having a true justified belief, because his belief is only accidentally true.¹⁵

Again, two different standards of justification motivate the case. One justificatory standard is used to claim that Henry has a justified, true belief – the immediate justification conferred on perceptual beliefs. Another justificatory standard is used to claim that Henry's belief is accidental – the low probability of randomly picking the real barn in such a district. Thus, while Henry may be justified in his belief as to what he sees, he is not justified in his belief as to the object he randomly picks out. Again, the latter justificatory standard prevails here, since Henry's belief as to the object he randomly picks is at issue, not his belief as to what he sees. That is, Henry is not justified in believing that the object he points to is a barn even though

he is justified in believing that he sees a barn. Thus, this case also equivocates on the justification condition, and so may be rejected.¹⁶

We now have non-epistemological approaches to these problem cases. Unfortunately, this implies that there is no epistemological connection between the cases. We can only offer an historical explanation for their existence: Gettier's original cases were widely understood as revealing a problem with justification. Once the philosophical community became generally convinced that there is such a problem, various attempts to bolster or amend justification spread through the literature. But since most philosophers were already convinced that there is a problem with justification, problem cases that equivocate on the justification condition, developed in response to each attempt at explicating justification, were seen as further examples of the problem Gettier discovered.

These examples do not undermine the traditional definition of knowledge. While they are supposed to show that there is a problem with justification, we have seen that they reveal no such problem. Contrary to the received view, the three standard conditions are sufficient for someone's knowing a given proposition. This does not negate the work that has been done to explicate justification. But that work need not answer the foregoing problem cases.

¹ (1963) "Is Justified True Belief Knowledge?" *Analysis* 23, 121-123.

² See below.

³ Some may object that I have usurped Gettier's right to stipulate the contents of Smith's belief as the author of this case. I have made no claim about the content of Smith's belief, only its truth conditions. One must separately assume a theory on which a proposition's content are given by its truth conditions in order to claim that the preceding argument dictates the contents of Smith's belief. Gettier does not explicitly make such an assumption, and I have tried to avoid it here. Of course, this argument presents interesting implications for the theory just mentioned, but those implications cannot be explored here.

⁴ See Myers, R. G., and Stern, K. (1973). "Knowledge Without Paradox." *Journal of Philosophy* 70: 147-60. See also Feldman, R. (1974). "An Alleged Defect in Gettier Counterexamples." *Australasian Journal of Philosophy* 52: 68-9.

⁵ Strictly speaking, this is only true if there are names for every object in the domain. Presumably, everyone in Smith's office has a name.

⁶ Due to the uniqueness clause in each formula, the disjunction of $\Phi, \Psi, \dots, \Omega$ will be an exclusive disjunction.

⁷ Lehrer, K. (1974). *Knowledge*. New York: Oxford University Press.

⁸ (Lehrer 1974, 20).

⁹ (Lehrer 1974, 20)

¹⁰ A similar argument may be used against Michael Williams' "candle" version of the perceptual belief argument. See Williams, M. (1996). *Unnatural Doubts: Epistemological Realism and the Basis of Skepticism*. Princeton: Princeton University Press, 94.

¹¹ Russell, Bertrand (1948). *Human Knowledge: Its Scope and Limits*. New York: Allen and Unwin.

¹² Scheffler, Israel (1965). *Conditions of Knowledge*. Chicago: Scott, Foresman.

¹³ *The Analysis of Knowing: A Decade of Research* (Princeton, NJ: Princeton University Press) 1983.

¹⁴ Goldman, Alvin (1976). "Discrimination and Perceptual Knowledge." *Journal of Philosophy* 73: 771 – 791. Reprinted in Goldman, A. (1992) *Liasons: Philosophy Meets the Cognitive and Social Sciences*. Cambridge MA: MIT Press. In the reprinted edition, Goldman adds a footnote (1) wherein he attributes this example to Carl Ginet, but Mr. Ginet seems not to have published it himself.

¹⁵ While Goldman uses this example in his (1976) to extend his externalism, here I merely note it as a proposed counterexample to the traditional analysis of knowledge.

¹⁶ Strictly speaking, the fact that the case equivocates between the two standards is reason enough to reject it as a counterexample.